IILX - The Implied Impermanent Loss Index

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Abstract

This white paper introduces IILX, the Derive.xyz Implied Impermanent Loss Index, a live market-based measure of impermanent loss in decentralized liquidity provision. The current methodology is designed for liquidity provision in pools with a token-stablecoin pair, such as ETH-stablecoin, and more generally for any underlying asset with a sufficiently liquid listed option market. The index uses option prices observed on Derive.xyz to infer the risk-neutral impermanent loss for liquidity providers in automated market makers under both concentrated-liquidity and full-range constant-product designs. Using SVI-parameterized implied volatility surfaces, we construct maturity-specific option smiles and evaluate option-based formulas for impermanent loss across a family of liquidity ranges around the forward price. The result is a forward-looking index over horizon and liquidity range width that can be used for monitoring, risk management, and comparative analysis of liquidity-provision risk in decentralized markets.

1 Introduction

Liquidity provision on decentralized exchanges generates fee income but also exposes liquidity providers (LPs) to impermanent loss when relative prices change. As arbitrage trades rebalance the pool after price changes, the LP's inventory shifts away from the composition of a passive buy-and-hold portfolio, causing the value of the liquidity position to diverge from the value of simply holding the underlying assets. In economic terms, liquidity provision is therefore closely related to a short exposure to relative-price variance.

This exposure becomes especially important in concentrated-liquidity AMMs such as Uniswap v3. Unlike full-range liquidity provision, concentrated liquidity allows LPs to choose a finite price interval over which their capital is active. Narrower ranges increase capital efficiency and fee intensity near the prevailing price, but they also amplify exposure to adverse price moves and out-of-range rebalancing. Wider ranges provide more protection against large price changes, but at the cost of lower fee concentration. Concentrated liquidity thus transforms impermanent-loss exposure into a design choice.

These features motivate a forward-looking, market-based measure of impermanent loss. Realized impermanent loss can be computed ex post from spot-price paths, but risk management, monitoring, and live market surveillance require an ex ante measure that updates as market conditions change. The purpose of the Derive Implied Impermanent Loss Index (IILX) is to provide such a measure. Using option prices observed on Derive.xyz, the index extracts the market-implied cost of liquidity provision across maturities and range specifications. The current implementation is intended for token-stablecoin liquidity provision, with ETH-stablecoin as the primary example. More generally, the same methodology applies to any underlying asset for which a sufficiently liquid option surface exists, allowing the risk-neutral distribution of future prices to be inferred from market data.

Our methodology proceeds in three steps. First, we recover maturity-specific implied volatility smiles from SVI-parametrized option surfaces [Gatheral, 2006]. Second, we convert those smiles into risk-neutral option prices using the Black-76 formula for the forward [Black, 1976].

Third, we evaluate option-based valuation formulas for impermanent loss under both concentrated-liquidity and full-range constant-product designs ([Papanicolaou, Schönleber, and Li, 2025, Papanicolaou, Schönleber, and Alberici, 2026]). The resulting index is a live, forward-looking measure of liquidity-provision risk that can be monitored across horizons and corridor widths.

The goal of this white paper is to document the construction of IILX and explain its economic interpretation. In doing so, it provides a transparent methodology for measuring the option-implied cost of decentralized liquidity provision and a framework for comparing concentrated-liquidity risk across market states.

2 Economic Setting

We consider liquidity provision between a risky asset and a stablecoin, with ETH-stablecoin as the leading example. The stablecoin serves as the numeraire, while the listed option surface is taken on the risky asset. Let a concentrated-liquidity position [Adams, Zinsmeister, Salem, Keefer, and Robinson, 2021] be defined over a price interval

$$[S_\ell, S_u], \qquad 0 < S_\ell < S_u.$$

The LP supplies liquidity to an automated market maker whose value function is nonlinear in the terminal asset price. Inside the band, the position continuously rebalances between the two assets; outside the band, the LP is effectively fully invested in one side of the pair.

The central object of interest is the loss of the LP portfolio relative to a passive benchmark that holds the corresponding inventory without rebalancing. This relative loss is path-dependent ex post, but under suitable option-replication arguments, it can be linked to the risk-neutral distribution of future prices and therefore inferred from option markets.

We focus on symmetric corridors around the forward price F_τ for a given maturity τ. For a corridor parameter $\alpha \in (0, 1)$, define

$$R_\alpha^- = (1-\alpha)F_\tau, \qquad R_\alpha^+ = (1+\alpha)F_\tau. \tag{1}$$

These bounds define the lower and upper edges of the liquidity range for the implied calculation.

3 Notation

We use the following notation throughout, omitting the time subscript t for simplicity. Unless otherwise stated, all calculations are understood as snapshots taken at a fixed point in time.

Symbol	Meaning
S	Spot price of the risky underlying
τ	Target maturity (for example, 30 days)
F_τ	Forward price of the risky underlying for the target maturity
$O(K, \tau)$	Out-of-the-money (OTM) option price at strike K and maturity τ
R_α^-, R_α^+	Lower and upper corridor boundaries
$\sigma_{\text{imp}}(K, \tau)$	Black-76 implied volatility at strike K and maturity τ
SVI	Stochastic Volatility Inspired parameterization

4 SVI-Based Implied Volatility Surface Construction

4.1 SVI total variance

For each snapshot date and maturity (τ), the code uses the Stochastic Volatility Inspired (SVI) parameterization Gatheral [2004] of the implied volatility smile, for which the parameters,

$$(a, b, \rho, m, \sigma) \ ,$$

are estimated from Derive.xyz option market data for the risky underlying asset, such as ETH, together with a forward F and a reference maturity τ. More generally, the same construction applies to any underlying asset with a sufficiently liquid option surface. For log-moneyness

$$k = \log(K/F_\tau),$$

the SVI total variance is written as

$$w(k) = a + b\left(\rho(k-m) + \sqrt{(k-m)^2 + \sigma^2}\right). \tag{2}$$

Implied volatility is then obtained as

$$\sigma_{\mathrm{imp}}(K,\tau) = \sqrt{\frac{w(k)}{\tau}}. \tag{3}$$

In practice, log-moneyness is clipped before evaluating the SVI smile to stabilize the tails. Writing

$$k = \log(K/F_\tau), \qquad k_{\mathrm{static}} = \sqrt{\max(0, a + b\sigma)},$$

the code truncates k to the interval $[-c\,k_{\mathrm{static}}, c\,k_{\mathrm{static}}]$, where $c = 3$. The resulting SVI total variance is then capped from above to avoid explosive tail values, with caps of 20. These adjustments are numerical safeguards intended to stabilize smile construction rather than additional modeling assumptions.

4.2 Smile construction on a moneyness grid

For each maturity slice, the smile is evaluated on a fixed grid of forward moneyness values

$$m = \frac{K}{F_\tau}.$$

Given a moneyness grid $\{m_i\}_{i=1}^N$, strikes are defined as

$$K_i = F_\tau\, m_i,$$

and the SVI formula is used to obtain $\sigma_{\mathrm{imp}}(K_i,\tau)$.

The implementation defaults to a grid spanning roughly $m \in [0.3, 2.0]$, which is then later clipped in delta space to avoid relying excessively on ill-behaved deep-tail extrapolation.

4.3 Interpolation across maturities

The target maturity may not coincide exactly with an observed SVI slice. In that case, we locate the nearest bracketing maturities $\tau_{\mathrm{low}} < \tau^\star < \tau_{\mathrm{high}}$ and interpolate in total variance.

Let

$$w_{\mathrm{low}}(K) = \sigma^2_{\mathrm{low}}(K)\tau_{\mathrm{low}}, \qquad w_{\mathrm{high}}(K) = \sigma^2_{\mathrm{high}}(K)\tau_{\mathrm{high}}.$$

Then the target total variance is

$$w^\star(K) = w_{\mathrm{low}}(K) + \lambda\left(w_{\mathrm{high}}(K) - w_{\mathrm{low}}(K)\right), \qquad \lambda = \frac{\tau^\star - \tau_{\mathrm{low}}}{\tau_{\mathrm{high}} - \tau_{\mathrm{low}}}. \tag{4}$$

The target implied volatility is then

$$\sigma^\star(K,\tau^\star) = \sqrt{\frac{w^\star(K)}{\tau^\star}}. \tag{5}$$

The forward used for the target smile is interpolated linearly between the two maturity forwards.

4.4 Delta filtering

After constructing the interpolated smile, we compute Black-76 forward deltas [Black, 1976]

$$d_1 = \frac{-\log(K/F) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}, \tag{6}$$

with call and put deltas

$$\Delta_{\text{call}} = N(d_1), \qquad \Delta_{\text{put}} = N(d_1) - 1.$$

The smile is then restricted to a moderate delta range, such as

$$0.01 \le \Delta_{\text{call}} \le 0.99, \qquad -0.99 \le \Delta_{\text{put}} \le -0.01.$$

This ensures that the pricing integrals are evaluated on a numerically reliable support.

5 The Implied Impermanent Loss

In what follows, the option-implied quantities are constructed from the option surface of the risky asset in the risky-asset/stablecoin pair.

5.1 Implied Impermanent Loss for v3

In this section, v3 refers to the concentrated-liquidity design introduced by Uniswap v3, in which liquidity is supplied over a finite price interval rather than across the full price range. For a symmetric corridor defined by

$$R_\alpha^- = (1-\alpha)F, \qquad R_\alpha^+ = (1+\alpha)F.$$

The index is computed across a family of corridor widths,

$$\alpha \in \{0.10, 0.15, 0.20, \dots, 0.90\},$$

thereby producing a surface over the maturity and range width. We compute the implied impermanent loss via the integral

$$\text{IILX}(\alpha,\tau) = \frac{1}{2\tau}\int_{R_\alpha^-}^{R_\alpha^+} \frac{O(K,\tau)}{K^{3/2}\left(2\sqrt{K} - \sqrt{R_\alpha^-} - \frac{K}{\sqrt{R_\alpha^+}}\right)}\, dK. \tag{7}$$

This expression can be interpreted as the risk-neutral expectation of the relative underperformance of a concentrated-liquidity position over the range $[R_\alpha^-, R_\alpha^+]$, represented through a weighted strip of OTM options. The derivations follow [Papanicolaou, Schönleber, and Li, 2025, Papanicolaou, Schönleber, and Alberici, 2026]. In practice, the integral is approximated numerically on a strike grid over the corridor using interpolated implied volatilities and Black-76 option prices [Black, 1976], see Appendix Appendix E.

5.2 Implied Impermanent Loss for V2

In this section, V2 refers to the constant-product liquidity provision design associated with Uniswap v2, where liquidity is supplied across the full price range. The corresponding IILX measure is obtained from the option surface through

$$\text{IILX}(v2,\tau) = \frac{1}{4\tau}\int \frac{O(K,\tau)}{K^2}\, dK. \tag{8}$$

6 Conclusion

This paper presents a practical and theoretically grounded framework for measuring impermanent loss using option markets. Starting from SVI-implied volatility surfaces, we recover maturity-specific smiles and evaluate a corridor-based integral that yields an IILX measure for concentrated liquidity provision.

The result is a unified panel of forward-looking liquidity-risk measures. This framework is suitable for empirical analysis, dashboarding, and strategy design, and can serve as a building block for richer LP valuation models that incorporate fees, dynamic range management, and market microstructure effects.

Below is a self-contained appendix section using the whitepaper's notation. It proves equations (7) and (8).

Appendix A Derivation of the IILX Formula

This appendix derives the option-replication formula in equation (7). We work in units of the stablecoin and consider a fixed concentrated-liquidity range

$$[R_\alpha^-, R_\alpha^+], \qquad 0 < R_\alpha^- < F_\tau < R_\alpha^+.$$

Appendix A.1 Risk-neutral price dynamics

Let S_t denote the price of the risky asset in stablecoin units. In the current implementation, the interest rate is set to zero. Consequently, $F_\tau = S_0$, and under the risk-neutral measure $\mathbb{Q}$,

$$dS_t = S_t \sigma_t \, dW_t^{\mathbb{Q}}, \qquad d\langle S \rangle_t = S_t^2 \sigma_t^2 \, dt, \tag{9}$$

where σ_t may be stochastic and adapted to the underlying filtration. We assume that S_t has continuous paths and that the stochastic integrals below are true martingales.

For nonzero interest rates, the same argument can be formulated in forward units. Equivalently, discounted Black–76 option prices can be converted into undiscounted prices by dividing by the discount factor, as described in the Black–76 appendix.

Appendix A.2 Value and curvature of a concentrated-liquidity position

For notational convenience, define

$$D_\alpha(s) = 2\sqrt{s} - \sqrt{R_\alpha^-} - \frac{s}{\sqrt{R_\alpha^+}}. \tag{10}$$

For $s \in [R_\alpha^-, R_\alpha^+]$, the inventories of a position with liquidity scale $\tilde{L}$ are

$$N_1(s) = \tilde{L} \left(\frac{1}{\sqrt{s}} - \frac{1}{\sqrt{R_\alpha^+}} \right), \tag{11}$$

$$N_2(s) = \tilde{L} \left(\sqrt{s} - \sqrt{R_\alpha^-} \right). \tag{12}$$

The position value in stablecoin units is therefore

$$\begin{aligned} V_P(s; R_\alpha^-, R_\alpha^+) &= sN_1(s) + N_2(s) \\ &= \tilde{L} \left(2\sqrt{s} - \sqrt{R_\alpha^-} - \frac{s}{\sqrt{R_\alpha^+}} \right) \\ &= \tilde{L} D_\alpha(s). \end{aligned} \tag{13}$$

Differentiating inside the active range gives

$$\frac{\partial V_P}{\partial s}(s) = \tilde{L}\left(\frac{1}{\sqrt{s}} - \frac{1}{\sqrt{R_\alpha^+}}\right) = N_1(s), \tag{14}$$

$$\frac{\partial^2 V_P}{\partial s^2}(s) = -\frac{\tilde{L}}{2s^{3/2}}. \tag{15}$$

Outside the active range, equation (39) is either linear or constant in s. Hence,

$$\frac{\partial^2 V_P}{\partial s^2}(s) = -\frac{\tilde{L}}{2s^{3/2}}\mathbf{1}_{\{R_\alpha^- \le s \le R_\alpha^+\}}. \tag{16}$$

The first derivative of V_P is continuous at both corridor boundaries. Consequently, the generalized Itô formula does not produce local-time terms at R_α^- or R_α^+.

Appendix A.3 Instantaneous impermanent loss

Suppose that, over an infinitesimal interval, the LP instead held its current token inventory fixed. In stablecoin units, the change in the value of that frozen inventory would be

$$N_1(S_t)\,dS_t = \frac{\partial V_P}{\partial s}(S_t)\,dS_t.$$

By Itô's formula, the change in the AMM position value is

$$dV_P(S_t) = \frac{\partial V_P}{\partial s}(S_t)\,dS_t + \frac{1}{2}\frac{\partial^2 V_P}{\partial s^2}(S_t)\,d\langle S\rangle_t. \tag{17}$$

The absolute loss generated by AMM rebalancing is therefore

$$\begin{aligned} dL_t^{\text{abs}} &= N_1(S_t)\,dS_t - dV_P(S_t) \\ &= -\frac{1}{2}\frac{\partial^2 V_P}{\partial s^2}(S_t)\,d\langle S\rangle_t. \end{aligned} \tag{18}$$

Using equations (9) and (16),

$$\begin{aligned} dL_t^{\text{abs}} &= -\frac{1}{2}\left(-\frac{\tilde{L}}{2S_t^{3/2}}\right) S_t^2\sigma_t^2\mathbf{1}_{\{R_\alpha^- \le S_t \le R_\alpha^+\}}dt \\ &= \frac{\tilde{L}}{4}\sqrt{S_t}\sigma_t^2\mathbf{1}_{\{R_\alpha^- \le S_t \le R_\alpha^+\}}dt. \end{aligned} \tag{19}$$

Dividing by the current LP position value $V_P(S_t) = \tilde{L}D_\alpha(S_t)$ gives the instantaneous loss:

$$d\mathrm{IL}_t^{\text{v3}} = \frac{\sqrt{S_t}\sigma_t^2}{4D_\alpha(S_t)}\mathbf{1}_{\{R_\alpha^- \le S_t \le R_\alpha^+\}}dt. \tag{20}$$

Integrating over the interval $[0, \tau]$,

$$\mathrm{IL}_\tau^{\text{v3}} = \frac{1}{4}\int_0^\tau \frac{\sqrt{S_t}\sigma_t^2}{D_\alpha(S_t)}\mathbf{1}_{\{R_\alpha^- \le S_t \le R_\alpha^+\}}dt. \tag{21}$$

Appendix A.4 Construction of the replicating option strip

Define the strike-weight function

$$w_\alpha(K) = \frac{\mathbf{1}_{\{R_\alpha^- \le K \le R_\alpha^+\}}}{2K^{3/2} D_\alpha(K)}. \tag{22}$$

Now define the terminal payoff

$$\begin{aligned} G_\alpha(x) = &\int_{R_\alpha^-}^{F_\tau} w_\alpha(K)(K-x)^+ \, dK \\ &+ \int_{F_\tau}^{R_\alpha^+} w_\alpha(K)(x-K)^+ \, dK. \end{aligned} \tag{23}$$

Thus, the first integral contains OTM puts and the second contains OTM calls. For every fixed strike K,

$$\frac{\partial^2}{\partial x^2}(K-x)^+ = \delta_K(x), \qquad \frac{\partial^2}{\partial x^2}(x-K)^+ = \delta_K(x),$$

where δ_K denotes the Dirac mass at K. It follows, in the distributional sense, that

$$G_\alpha''(x) = w_\alpha(x) = \frac{\mathbf{1}_{\{R_\alpha^- \le x \le R_\alpha^+\}}}{2x^{3/2} D_\alpha(x)}. \tag{24}$$

Because all options in equation (23) are OTM at the anchor $F_\tau = S_0$,

$$G_\alpha(S_0) = 0, \qquad G_\alpha'(S_0) = 0. \tag{25}$$

Appendix A.5 Application of Itô's formula

Applying the generalized Itô formula to $G_\alpha(S_t)$ gives

$$\begin{aligned} G_\alpha(S_\tau) = G_\alpha(S_0) &+ \int_0^\tau G_\alpha'(S_t) \, dS_t \\ &+ \frac{1}{2}\int_0^\tau G_\alpha''(S_t) \, d\langle S\rangle_t. \end{aligned} \tag{26}$$

Using equations (9), (24), and (25),

$$\begin{aligned} G_\alpha(S_\tau) = &\int_0^\tau G_\alpha'(S_t) \, dS_t \\ &+ \frac{1}{4}\int_0^\tau \frac{\sqrt{S_t}\sigma_t^2}{D_\alpha(S_t)} \mathbf{1}_{\{R_\alpha^- \le S_t \le R_\alpha^+\}} dt. \end{aligned} \tag{27}$$

Comparing this expression with equation (21), we obtain

$$G_\alpha(S_\tau) = \int_0^\tau G_\alpha'(S_t) \, dS_t + \mathrm{IL}_\tau^{\mathrm{v3}}. \tag{28}$$

Since S_t is a martingale under $\mathbb{Q}$, the stochastic integral has zero expectation. Therefore,

$$\mathbb{E}^{\mathbb{Q}}\left[\mathrm{IL}_\tau^{\mathrm{v3}}\right] = \mathbb{E}^{\mathbb{Q}}\left[G_\alpha(S_\tau)\right]. \tag{29}$$

Appendix A.6 Expression in terms of option prices

The undiscounted OTM option price is

$$O(K,\tau) = \begin{cases} \mathbb{E}^{\mathbb{Q}}[(K - S_\tau)^+], & K < F_\tau, \\ \mathbb{E}^{\mathbb{Q}}[(S_\tau - K)^+], & K \geq F_\tau. \end{cases} \tag{30}$$

When $r = 0$, this equals the observed Black–76 option price.

Taking expectations in equation (23) and using Tonelli's theorem,

$$\begin{aligned} \mathbb{E}^{\mathbb{Q}}[G_\alpha(S_\tau)] &= \int_{R_\alpha^-}^{F_\tau} w_\alpha(K)\mathbb{E}^{\mathbb{Q}}\left[(K - S_\tau)^+\right] dK \\ &\quad + \int_{F_\tau}^{R_\alpha^+} w_\alpha(K)\mathbb{E}^{\mathbb{Q}}\left[(S_\tau - K)^+\right] dK \\ &= \int_{R_\alpha^-}^{R_\alpha^+} w_\alpha(K) O(K,\tau)\, dK. \end{aligned} \tag{31}$$

Substituting the strike weight from equation (22),

$$\mathbb{E}^{\mathbb{Q}}\left[\mathrm{IL}_\tau^{\mathrm{v3}}\right] = \frac{1}{2}\int_{R_\alpha^-}^{R_\alpha^+} \frac{O(K,\tau)}{K^{3/2}\left(2\sqrt{K} - \sqrt{R_\alpha^-} - \dfrac{K}{\sqrt{R_\alpha^+}}\right)}\, dK. \tag{32}$$

Finally, dividing by the maturity τ, expressed in years, produces the annualized index:

$$\boxed{\mathrm{IILX}(\alpha,\tau) = \frac{1}{2\tau}\int_{R_\alpha^-}^{R_\alpha^+} \frac{O(K,\tau)}{K^{3/2}\left(2\sqrt{K} - \sqrt{R_\alpha^-} - \dfrac{K}{\sqrt{R_\alpha^+}}\right)}\, dK}. \tag{33}$$

This is equation (7).

Appendix A.7 Full-range limit

The v2 formula follows as a limiting case. Let

$$R_\alpha^- \downarrow 0, \qquad R_\alpha^+ \uparrow \infty.$$

For every fixed $K > 0$,

$$D_\alpha(K) = 2\sqrt{K} - \sqrt{R_\alpha^-} - \frac{K}{\sqrt{R_\alpha^+}} \longrightarrow 2\sqrt{K}. \tag{34}$$

Consequently,

$$K^{3/2} D_\alpha(K) \longrightarrow 2K^2.$$

Substituting this limit into the concentrated-liquidity formula gives

$$\begin{aligned} \mathrm{IILX}(v2,\tau) &= \frac{1}{2\tau}\int_0^\infty \frac{O(K,\tau)}{2K^2}\, dK \\ &= \boxed{\frac{1}{4\tau}\int_0^\infty \frac{O(K,\tau)}{K^2}\, dK}. \end{aligned} \tag{35}$$

This coincides with equation (8) and confirms the factor $1/2$ in the concentrated-liquidity formula.

Appendix B Core Computational Steps

For completeness, the computational workflow can be summarized as follows.

1. **Load market data:** SVI parameters, forward, spot, and maturity per snapshot.
2. **Select target maturity:** use exact maturity if available; otherwise, interpolate between the nearest lower and higher slices.
3. **Construct implied smile:** evaluate SVI on a fixed moneyness grid and calculate implied volatilities.
4. **Filter tail points:** retain only strikes whose deltas lie in a numerically stable range.
5. **Price options:** compute Black-76 call and put prices on the forward.
6. **Compute IILX:** numerically integrate the corridor formula for each α, and compute the v2 benchmark.

Appendix C Black-76 Pricing

Given the forward F of the risky underlying asset, strike K, maturity τ, discount factor $DF = e^{-r\tau}$, and implied volatility σ, the discounted Black-76 call and put prices are

$$C^{\text{disc}}(K) = DF\left(FN(d_1) - KN(d_2)\right), \tag{36}$$

$$P^{\text{disc}}(K) = DF\left(KN(-d_2) - FN(-d_1)\right), \tag{37}$$

where

$$d_1 = \frac{\log(F/K) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}, \qquad d_2 = d_1 - \sigma\sqrt{\tau}.$$

We denote the OTM option price by

$$O(K,\tau) = \begin{cases} P(K,\tau), & K < F, \\ C(K,\tau), & K \geq F. \end{cases} \tag{38}$$

In the empirical implementation considered here, $r = 0$, so discounted and undiscounted prices coincide. More generally, when needed, discounted [Black, 1976] prices can be converted into undiscounted prices by dividing by the discount factor DF.

Appendix D Realized Impermanent Loss (RIL)

To compare implied and realized measures, we construct an RIL series from spot prices sampled at regular intervals, typically hourly.

Let S_0 denote the initial spot level over one step and S_1 the next observation. For a corridor parameter α, define

$$S_\ell = (1-\alpha)S_0, \qquad S_u = (1+\alpha)S_0.$$

These bounds re-center the LP range around the current spot.

We calculate the terminal LP value as

$$V_P(S_1; S_\ell, S_u) = \tilde{L}\begin{cases} S_1\left(\frac{1}{\sqrt{S_\ell}} - \frac{1}{\sqrt{S_u}}\right), & S_1 < S_\ell, \\ 2\sqrt{S_1} - \sqrt{S_\ell} - \frac{S_1}{\sqrt{S_u}}, & S_\ell \leq S_1 < S_u, \\ \sqrt{S_u} - \sqrt{S_\ell}, & S_1 \geq S_u, \end{cases} \tag{39}$$

where $\tilde{L}$ is a scaling constant for liquidity ([Heimbach, Schertenleib, and Wattenhofer, 2022]).

The buy-and-hold benchmark is evaluated as

$$V_H(S_0, S_1; S_\ell, S_u) = \tilde{L} \begin{cases} S_1 \left(\dfrac{1}{\sqrt{S_\ell}} - \dfrac{1}{\sqrt{S_u}} \right), & S_0 < S_\ell, \\ \dfrac{S_0 + S_1}{\sqrt{S_0}} - \sqrt{S_\ell} - \dfrac{S_1}{\sqrt{S_u}}, & S_\ell \le S_0 < S_u, \\ \sqrt{S_u} - \sqrt{S_\ell}, & S_0 \ge S_u. \end{cases} \tag{40}$$

The one-step RIL is computed as

$$\mathrm{RIL}^{\mathrm{step}}_{t \to t+1} = -\frac{V_P - V_H}{V_H}. \tag{41}$$

For a target horizon of m days and an observation frequency with annualization constant A (e.g. $A = 24$ for hourly data), the rolling window length is $W = mA$.

The RIL is then calculated as the annualized rolling mean

$$\mathrm{RIL}(\alpha, m)_t = 365A \cdot \frac{1}{W} \sum_{j=0}^{W-1} \mathrm{RIL}^{\mathrm{step}}_{t-j \to t-j+1}, \tag{42}$$

subject to a minimum-data requirement in the rolling window.

For v2, the RIL is

$$\mathrm{RIL}(v2, m)_t = \frac{1}{8} \cdot 365A \cdot \mathrm{Var}_t(\Delta S/S), \tag{43}$$

where the variance is computed over the same rolling window.

Appendix E Numerical Approximation of the Integrals

In the implementation, the option-integral formulas are evaluated numerically on a discrete strike grid using a trapezoidal-type quadrature rule. Let

$$K_1 < K_2 < \cdots < K_n$$

denote the strike grid. The integration weights are defined by

$$\Delta K_1 = K_2 - K_1, \qquad \Delta K_n = K_n - K_{n-1}, \qquad \Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}, \quad i = 2, \dots, n-1. \tag{44}$$

When the grid is equally spaced, this coincides with the standard trapezoidal rule.

For the concentrated-liquidity index, define the integrand

$$g(K; \alpha, \tau) = \frac{O(K, \tau)}{K^{3/2} \left(2\sqrt{K} - \sqrt{R_\alpha^-} - \frac{K}{\sqrt{R_\alpha^+}} \right)}. \tag{45}$$

The strike grid is taken over the corridor

$$K_i \in [R_\alpha^-, R_\alpha^+], \qquad i = 1, \dots, n,$$

typically as an equally spaced grid. The index is then approximated by

$$\mathrm{IILX}(\alpha, \tau) \approx \frac{1}{2\tau} \sum_{i=1}^{n} \Delta K_i \, g(K_i; \alpha, \tau). \tag{46}$$

Equivalently, writing the option prices explicitly,

$$\mathrm{IILX}(\alpha,\tau) \approx \frac{1}{2\tau}\sum_{i=1}^{n}\Delta K_i \frac{O(K_i,\tau)}{K_i^{3/2}\left(2\sqrt{K_i}-\sqrt{R_\alpha^-}-\frac{K_i}{\sqrt{R_\alpha^+}}\right)}. \tag{47}$$

Here, $O(K_i,\tau)$ is computed from the interpolated implied-volatility smile using Black-76 option pricing on the forward [Black, 1976].

For the full-range constant-product benchmark, define

$$h(K;\tau) = \frac{O(K,\tau)}{K^2}. \tag{48}$$

Let

$$K_1 < K_2 < \cdots < K_m$$

denote the strike grid over the available support of the implied-volatility smile. Then

$$\mathrm{IILX}(v2,\tau) \approx \frac{1}{4\tau}\sum_{i=1}^{m}\Delta K_i\, h(K_i;\tau) = \frac{1}{4\tau}\sum_{i=1}^{m}\Delta K_i\,\frac{O(K_i,\tau)}{K_i^2}. \tag{49}$$

In the implementation, implied volatilities are first evaluated on the strike grid by interpolation of the SVI-implied smile. Option prices $O(K_i,\tau)$ are then computed pointwise from those implied volatilities. The numerical integrals in (47) and (49) are finally obtained by weighted summation using the grid increments in (44).

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